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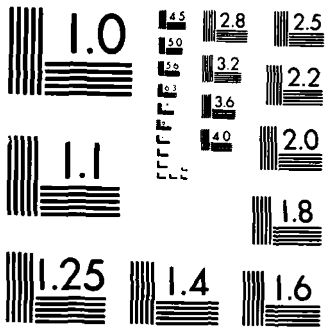

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A PRELIMINARY REPORT
ON
SOME RECENT RESULTS IN BORN INVERSION

by

Jack K. Cohen, Frank G. Hagin, and Norman Bleistein

Partially supported by the Consortium Project of the Center for Wave Phenomena and by the Selected Research Opportunities Program of the Office of Naval Research

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1. INTRODUCTION

The research group at the Center for Wave Phenomena has been developing inversion algorithms for progressively more complex background velocities and source-receiver configurations. In each extension, the crucial issue has been the determination of certain properties of a matrix involving derivatives of the travel time(s).

This report discusses and extends a recent result along these lines which appeared in a paper by Gregory Beylkin (1985). Beylkin reduces the problem to consideration of a single canonical determinant, h, (defined below) and then assumes that this determinant does not vanish. With this assumption, he establishes a very general inversion result.

Obviously, then, future theoretical research will focus on the evaluation of h and on establishing conditions for the non-vanishing of Beylkin's determinant, h, and on dealing with the phenomena arising when it does vanish.

The more serious cases of vanishing h are due to phenomena such as caustics in the incident ray field. However, Beylkin's h will also vanish in the case of insufficient data. Thus, for example, we rule out such obvious impossibilities as obtaining a 3-D inversion from a single line of data, etc. In the case of insufficient data, one must make additional assumptions about the subsurface geometry consistent with the given data configuration. In the example just mentioned, one would be forced to assume that the subsurface was independent of the direction orthogonal to the line of observations. One could then either use the 2D wave equation (i.e. assume line sources) as in classical migration, (Schneider, 1978), (Stolt, 1978), or, preferably, continue to use the 3D wave equation (i.e. use point sources), (Cohen and Bleistein, 1979). This latter approach is known as the

2.5D geometry and is expounded in detail in a recent Center for Wave Phenomena Research Report (Bleistein, 1984a).

Beylkin's paper uses powerful mathematical tools, such as the notions of pseudo-differential operators, generalized Radon transforms, and generalized back projections. Moreover, Beylkin frames his work in an N-dimensional space.

Here, we dispense with this mathematical machinery and for convenience confine ourselves to the 3D case (and its 2.5D specialization). We are able to expound Beylkin's results by an approach similar to that presented earlier in (Cohen and Hagin, 1985). However, we do not attempt to rigorously prove our results, but instead content ourselves with an intuitive derivation and the citing of Beylkin's theorem. We remark in passing that we are contemplating the possibility of developing a classical proof of Beylkin's result.

We have established significant simplifications of Beylkin's result for the following cases of propagation governed by the acoustic wave equation:

(1) The zero-offset case with a general $c(x,y,z)$ reference velocity. This work is described in the present report. See also (Cohen and Hagin, 1985) for the special case of a $c(z)$ reference velocity. Here, in all but the case of a constant reference, one has to exclude "turned rays" in order to guarantee that h (or its equivalent in earlier approaches) does not vanish. It is easy to build this restriction into the zero-offset algorithm (Sumner, 1985), but only at the cost of excluding the imaging of reflectors "from below". At the present time, the Center for Wave Phenomena is investigating the use of turned rays in the $c(z)$ case.

(2) The common source offset gather and the common receiver offset gather with general $c(x,y,z)$ reference velocity. This work is also described below.

(3) The case of common offset with a constant reference velocity. Here, an explicit inversion formula has been obtained (Sullivan and Cohen, 1985). We remark that the derivation of this result does not proceed from the Born approach described below, but instead starts from the Kirchhoff (high frequency) representation of the scattered field (Bleistein, 1984b). Work on coding and testing this algorithm is now under way.

II. The General Inversion Formula

We consider a completely arbitrary source-receiver configuration parametrized by two surface coordinates, ξ_1 and ξ_2. Vectors, $\underline{x}$ and $\underline{x}'$ denote arbitrary subsurface field points. Vector $\underline{x}_r$ denotes a generic source, while $\underline{x}_s$ denotes a generic receiver. Any relation between the sources and receivers is defined by the dependencies of $\underline{x}_r$ and $\underline{x}_s$ on the parameters ξ_1 and ξ_2:

$$\underline{x} = (x_1, x_2, x_3) \quad , \quad \underline{x}' = (x_1', x_2', x_3')$$
$$\underline{\xi} = (\xi_1, \xi_2) \; , \quad \underline{x}_s = \underline{x}_s(\underline{\xi}), \quad \underline{x}_r = \underline{x}_r(\underline{\xi}) \cdot \tag{1}$$

For example, if

$$\underline{x}_r(\underline{\xi}) = \underline{x}_s(\underline{\xi}) \quad , \tag{2}$$

then we have the case of zero offset. If

$$\underline{x}_s(\underline{\xi}) = \text{constant}, \tag{3}$$

then we have the common source configuration. If

$$\underline{x}_r(\underline{\xi}) = \underline{x}_s(\underline{\xi}) + 2\underline{d}, \quad \underline{d} = \text{constant}, \tag{4}$$

then we have the common offset case, etc.

We note that the above formulation of the source-receiver configuration is somewhat more general and inclusive than the formulation in Beylkin's paper. Our formulation admits the possibility of a curved observation surface, while he makes the usual assumption of observations on the flat observation plane $z = x_3 = 0$. We have included this mild generalization because it may aid in the development of pre-statics inversion algorithms (May and Covey, 1980). Also, Beylkin splits off his derivation of the common source result into a separate case from the case when the receiver locations depend on the source locations.

As expounded in several previous papers and reports, we make the following assumptions:

(1) the velocity, $v(x,y,z)$, is well approximated by a known reference velocity, $c(x,y,z)$, so that

$$\frac{1}{v^2(\underline{x})} = \frac{1}{c^2(\underline{x})}\,(1+\alpha(\underline{x})), \tag{5}$$

where $\alpha(x,y,z)$ is a perturbation correction,

(2) the seismic fields are governed with sufficient accuracy by the 3-D acoustic wave equation, and

(3) the seismic source can be reduced to an ideal 3-D point source, so that:

$$\nabla^2 u(t,\underline{x}) - \frac{1}{c^2(\underline{x})}\frac{\partial^2}{\partial t^2}u(t,\underline{x}) = -\delta(t)\delta(\underline{x}-\underline{x}_s) \ . \tag{6}$$

These assumptions and the application of Green's theorem lead to the linear integral equation,

$$D(\omega, \underline{x}_r, \underline{x}_s) \approx \omega^2 \iiint d^3x' \; \frac{G(\omega, \underline{x}', \underline{x}_s) G(\omega, \underline{x}', \underline{x}_r)}{c^2(\underline{x}')} \alpha(\underline{x}') \; . \tag{7}$$

Here, by the assumption that α in (5) is small, we have been able to approximate the field emanating from the sources by one governed by a wave equation with a velocity function equal to the reference velocity, c, instead the full velocity, v. Since our sources are point sources, the functions denoted by G in equation (7) are the impulse response or Green's functions for the wave equation with velocity $c(x, y, z)$. Equation (7) is an integral equation for the unknown velocity perturbation, $\alpha(x, y, z)$, with data being the observations, D, at the receivers, $\underline{x}_r$, due to the sources, $\underline{x}_s$.

We now exploit the fact that geophysical data resides in the high frequency regime (Bleistein, 1984b), so that we may replace the Green's functions in (7), by their WKBJ approximations,

$$G(\omega, \underline{x}, \underline{x}_0) \sim A(\underline{x}, \underline{x}_0) \, e^{i\omega\tau(\underline{x}, \underline{x}_0)} \quad , \tag{8}$$

where $\underline{x}_0$ is either $\underline{x}_s$ or $\underline{x}_r$ and the travel-time phase satisfies the eikonal equation,

$$\nabla\tau \cdot \nabla\tau = \frac{1}{c^2(\underline{x})} \quad , \tag{9}$$

while the amplitude satisfies the transport equation,

$$2\nabla\tau \cdot \nabla A + A\nabla^2\tau = 0 \quad . \tag{10}$$

Thus, we may rewrite (7) as

$$D(\omega, \underline{x}_r, \underline{x}_s) \sim \omega^2 \iiint d^3x' \, \frac{a(\underline{x}', \xi)}{c^2(\underline{x}')} \, e^{i\omega\Phi(\underline{x}', \xi)} \, \alpha(\underline{x}') \tag{11}$$

where $\Phi(\underline{x}, \xi)$ and $a(\underline{x}, \xi)$ are given by

$$\begin{aligned} \Phi(\underline{x}, \xi) &= \tau(\underline{x}, \underline{x}_s) + \tau(\underline{x}, \underline{x}_r) \quad , \\ a(\underline{x}, \xi) &= A(\underline{x}, \underline{x}_s) A(\underline{x}, \underline{x}_r) \quad . \end{aligned} \tag{12}$$

From the basic migration principle of "backward propagation", it is not hard to guess that the inversion operator for (11) will have the negative of the phase in (11). The correct amplitude is not as easy to guess and in our inversion equation,

$$\alpha(\underline{x}) \sim \iint d^2\xi \, \sqrt{g(\xi)} \, b(\underline{x}, \xi) \int d\omega \, F(\omega) \, e^{-i\omega\Phi(\underline{x}, \xi)} D(\omega, \underline{x}_r, x_s) \ , \tag{13}$$

we merely denote it by $b(\underline{x}, \xi)$, and deduce it below. In equation (13), $\sqrt{g}$ denotes the differential area element on the observation surface (unity for a plane) and $F(\omega)$ denotes a known high band pass filter which is included to honor the fact that the data is confined to the high frequency regime. Note that while a priori the unknown amplitude could depend on ω, we suppress this potential dependence because our prior inversion results suggest that $b(\underline{x}, \xi)$ is independent of ω to first order. The results to follow also confirms this assumption.

Inserting (11) into (13) gives an equation which maps $\alpha(\underline{x}')$ to $\alpha(\underline{x})$. Thus, since we have an integral over all $\underline{x}'$ space, the kernel must be the three-dimensional Dirac delta function. A slight rewriting of this fact gives

$$\iint d^2\xi \sqrt{g}\, a(\underline{x}',\xi)b(\underline{x},\xi) \cdot$$
$$\int d\omega\, \omega^2 F(\omega) e^{i\omega[\Phi(\underline{x}',\xi)-\Phi(\underline{x},\xi)]} \sim c^2(\underline{x})\delta(\underline{x}-\underline{x}') \ . \quad (14)$$

Our task is to solve (14) for the unknown amplitude, b. Then equation (13) will give the inversion algorithm for α.

Since we have used high frequency approximations, we cannot recover low frequency phenomena such as trend information. We can expect to recover information about the discontinuities (i.e. rapid changes) in the substructure.

The essence of Beylkin's result is that if we confine attention to the discontinuity structure of α, then in (14), we need keep only two terms of the Taylor series of $\Phi(\underline{x}',\xi)$ about $\underline{x}' = \underline{x}$; and similarly we need keep only one term of the expansion of $a(\underline{x}',\xi)$. In order to establish these facts rigorously Beylkin uses powerful results in the theory of generalized Radon transforms. However, noting that the result on the left must involve a Dirac delta function acting at $\underline{x}' = \underline{x}$, these approximations are intuitively reasonable. Thus in (14), we use the approximations

$$\Phi(\underline{x}',\xi) \approx \Phi(\underline{x},\xi) + \nabla\Phi(\underline{x},\xi)\cdot(\underline{x}'-\underline{x}) \quad ,$$
$$a(\underline{x}',\xi) \approx a(\underline{x},\xi) \quad (15)$$

to obtain

$$\iint d^2\xi \sqrt{g}\, a(\underline{x},\xi)b(\underline{x},\xi) \cdot$$
$$\int d\omega\, \omega^2 F(\omega) e^{i\omega\nabla\Phi(\underline{x},\xi)\cdot(\underline{x}-\underline{x}')} \approx c^2(\underline{x})\delta(\underline{x}-\underline{x}') \quad . \quad (16)$$

Following Beylkin, we make the change of variables,

$$\underline{k} = \omega\nabla\Phi(\underline{x},\xi) \quad (17)$$

from ω, ξ_1, ξ_2 to k_1, k_2, k_3 (while viewing $\underline{x}$ as a "parameter"). Since the

ω dependence on the Jacobian of this transformation is just a power, Beylkin defines the scaled Jacobian, h, by

$$h(\underline{x},\underline{\xi}) = \begin{vmatrix} \frac{\partial \Phi}{\partial x_1} & \frac{\partial \Phi}{\partial x_2} & \frac{\partial \Phi}{\partial x_3} \\ \frac{\partial^2 \Phi}{\partial x_1 \partial \xi_1} & \frac{\partial^2 \Phi}{\partial x_2 \partial \xi_1} & \frac{\partial^2 \Phi}{\partial x_3 \partial \xi_1} \\ \frac{\partial^2 \Phi}{\partial x_1 \partial \xi_2} & \frac{\partial^2 \Phi}{\partial x_2 \partial \xi_2} & \frac{\partial^2 \Phi}{\partial x_3 \partial \xi_2} \end{vmatrix} \quad . \tag{18}$$

We now rewrite (16) as

$$\iiint d^3k \, \frac{\sqrt{g}}{|h(\underline{x},\underline{\xi})|} \, a(\underline{x},\underline{\xi}) b(\underline{x},\underline{\xi}) e^{i\underline{k}\cdot(\underline{x}-\underline{x}')} \approx c^2(\underline{x}) \delta(\underline{x}-\underline{x}') \quad . \tag{19}$$

From (19) and the classical Fourier transform completeness relation, it is clear how to choose the inversion amplitude b:

$$b(\underline{x},\underline{\xi}) = \frac{1}{8\pi^3} \frac{c^2(x) \, |h(\underline{x},\underline{\xi})|}{\sqrt{g} \, a(\underline{x},\underline{\xi})} \quad . \tag{20}$$

Inserting this result into (13) gives the inversion result:

$$\alpha(\underline{x}) \sim \frac{c^2(x)}{8\pi^3} \iint d^2\xi \, \frac{|h(x,\xi)|}{a(\underline{x},\underline{\xi})} \int d\omega \, e^{-i\omega\Phi(\underline{x},\underline{\xi})} D(\omega,\underline{x}_r,\underline{x}_s) \tag{21}$$

where Φ and a are defined above in (12).

It is intriguing to note that if we define the kernel,

$$K(\omega,\underline{x},\underline{x}_s,\underline{x}_r) = \frac{\omega^2 G(\omega,\underline{x},\underline{x}_s) G(\omega,\underline{x},\underline{x}_r)}{c^2(\underline{x})} \tag{22}$$

then we may state our integral equation and its inversion as

$$D(\omega, \underline{x}_r, x_s) \approx \iiint d^3x \; K(\omega, \underline{x}, \underline{x}_s, \underline{x}_r)\,\alpha(\underline{x}) \tag{23}$$

and

$$\alpha(\underline{x}) \approx \frac{1}{8\pi^3} \iiint d^3k \; \frac{1}{K(\omega, \underline{x}_s, \underline{x}_r, \underline{x})} D(\omega, \underline{x}_r, x_s) \quad . \tag{24}$$

In the last equation, we must regard ω as a function of $\underline{x}$ and $\underline{k}$ as defined by the change of variables (17).

III. The Zero-Offset Configuration

In the case of zero-offset, we can find a representation of the Jacobian h in terms of the rays used to construct the Green's function. Using the notation $\underline{x}_0$, as in (2), for the zero-offset source-receiver point we have

$$h(\underline{x},\underline{\xi}) = \begin{vmatrix} 2\nabla\tau(\underline{x},\underline{x}_0) \\ 2\,\frac{\partial}{2\xi_1}\,\nabla\tau(\underline{x},\underline{x}_0) \\ 2\,\frac{\partial}{\partial\xi_2}\,\nabla\tau(\underline{x},\underline{x}_0) \end{vmatrix} = 8 \begin{vmatrix} \underline{p}(\underline{x},\underline{x}_0) \\ \frac{\partial}{\partial\xi_1}\,\underline{p}(\underline{x},\underline{x}_0) \\ \frac{\partial}{\partial\xi_2}\,\underline{p}(\underline{x},\underline{x}_0) \end{vmatrix} \qquad (25)$$

where in the second equality, we have introduced

$$\underline{p}(\underline{x},\underline{x}_0) = \nabla\tau(\underline{x},\underline{x}_0) \quad , \qquad (26)$$

and where the $\underline{\xi}$ dependence in p comes from $\underline{x}_0 = \underline{x}_0(\underline{\xi})$.

From the eikonal equation (9), we have

$$\underline{p} \cdot \underline{p} = \frac{1}{c^2(\underline{x})} \quad . \qquad (27)$$

Since the speed, c, is independent of $\underline{\xi}$, we also derive from (27)

$$\underline{p} \cdot \frac{\partial}{\partial\xi_i}\,\underline{p} = 0, \qquad i = 1,2. \qquad (28)$$

We now multiply the third column of h by p_3 and compensate with the reciprocal of p_3 outside the determinant, then on multiplying the first two columns by respectively p_1 and p_2, and adding these to the third column, we obtain

$$h(\underline{x},\xi) = \frac{8}{p_3} \begin{vmatrix} p_1 & p_2 & 1/c^2 \\ \dfrac{\partial p_1}{\partial \xi_1} & \dfrac{\partial p_2}{\partial \xi_1} & 0 \\ \dfrac{\partial p_1}{\partial \xi_2} & \dfrac{\partial p_2}{\partial \xi_2} & 0 \end{vmatrix} \tag{29}$$

which can be written as

$$h(\underline{x},\xi) = \frac{8}{c^2(\underline{x})\,p_3(\underline{x},\underline{x}_0)} \;\frac{\partial(p_1,p_2)}{\partial(\xi_1,\xi_2)} \;. \tag{30}$$

Here we have introduced the Jacobian notation

$$\frac{\partial(p_1,p_2)}{\partial(\xi_1,\xi_2)} = \begin{vmatrix} \dfrac{\partial p_1}{\partial \xi_1} & \dfrac{\partial p_2}{\partial \xi_1} \\ \dfrac{\partial p_1}{\partial \xi_2} & \dfrac{\partial p_2}{\partial \xi_2} \end{vmatrix} \;. \tag{31}$$

The evaluation of this determinant is a key, and often difficult, step. In appendix A, we show that

$$\frac{\partial(p_1,p_2)}{\partial(\xi_1,\xi_2)} = -16\pi^2 A^2(\underline{x},\underline{x}_0)\,p_3(\underline{x},\underline{x}_0)\,\sqrt{g}\;\hat{\underline{n}}\cdot\underline{p}_0 \tag{32}$$

where $\underline{p}_0$ is the initial direction of the ray from $\underline{x}_0$ to $\underline{x}$ and $\hat{\underline{n}}$ is the downward normal to the data surface at $\underline{x}_0$. Thus the absolute value of h is given by

$$\left|h(\underline{x},\xi)\right| = \frac{8A^2(\underline{x},\underline{x}_0)}{c^2(\underline{x})} \cdot 16\pi^2\sqrt{g}\;\hat{\underline{n}}\cdot\underline{p}_0 \tag{33}$$

and the zero-offset inversion for general reference velocity can be expressed as

$$\alpha(\underline{x}) \sim \frac{16}{\pi} \iint d^2\xi \sqrt{g}\, \hat{n} \cdot \underline{p}_0 \int d\omega\, e^{-2i\omega\tau(\underline{x},\underline{x}_0)} D(\omega,\underline{x}_0) \quad . \tag{34}$$

This is a substantial simplification of the general result (21), since it obviates the necessity for computing the amplitude when tracing the rays between surface and field points. Note from appendix A that computation of the amplitude would, in turn, require computation of the ray Jacobian. The net result is that we save six equations along rays and have only to solve the remaining seven equations for $\underline{x}$, $\underline{p}$, and τ.

The special case of this result in which the reference speed is a function of depth only was derived in (Cohen and Hagin 1985). That result agrees with the present one aside from a slight change in notation. Of course the present result also agrees with the constant reference speed result (Bleistein, 1985).

There are two further issues to be addressed concerning the implementation of (34). These are the specialization to the case of a linear array (the 2.5 dimensional case mentioned in the introduction) and the fact that discerning discontinuities is made easier if one replaces α by the reflectivity function, β, (related to the normal derivative of α) (Bleistein, 1984b).

In the usual case of a data set collected on a linear array instead of a full 2D array, we cannot obtain a full 3D subsurface inversion. A model consistent with this restricted set of observations is one in which both the observation surface and the subsurface are assumed to be "cylindrical" with respect to x_2:

$$c = c(x_1, x_3) \quad , \quad \alpha = \alpha(x_1, x_3) \tag{35}$$

and that the observation surface had the special parametrization:

$$x_1 = x_1(\xi_1), \qquad x_2 = \xi_2, \qquad x_3 = x_3(\xi_1) \quad . \tag{36}$$

In the 2.5 case, it is possible to eliminate the integral over ξ_2 by the method of stationary phase. Carrying out this step and for simplicity further specializing to the case of linear observations,

$$x_1 = \xi_1, \qquad x_3 = 0 \quad , \tag{37}$$

we obtain the 2.5D result,

$$\alpha(x_1, x_3) \sim \frac{16}{\sqrt{\pi}} \int d\xi_1 \sqrt{|\sigma_f|}\, q_3 \int \frac{d\omega}{\sqrt{i\omega}} F(\omega)\, e^{-2i\omega\tau(x_1, x_3, \xi_1, 0)} D(\omega, \xi_1) \quad . \tag{38}$$

Here σ_f denotes the final value of the ray parameter σ for the ray from $\underline{x}_0$ to $\underline{x}$ (or equivalently from $\underline{x}$ to $\underline{x}_0$) and q_3 is the value of p_3 on the observation array (see (40) below). In the reference just cited, it is shown that we may use 2D calculations to compute the rays:

$$\frac{dx_1}{d\sigma} = p_1 \ , \quad x_1(0) = \xi_1$$

$$\frac{dx_3}{d\sigma} = p_3 \ , \quad x_3(0) = 0$$

$$\frac{dp_1}{d\sigma} = \frac{1}{2} \frac{\partial}{\partial x_1} \frac{1}{c^2(x_1, x_3)} \ , \quad p_1(0) = q_1 \tag{39}$$

$$p_3 = \sqrt{c^2 - p_1^2}$$

$$\frac{d\tau}{d\sigma} = \frac{1}{c^2} \ , \quad \tau(0) = 0$$

To apply (38), we fix the desired field point, (x_1, x_3) and a suitable aperture of observation points $(\xi_1, 0)$ on the observation array. Then we trace the ray from $(\xi_1, 0)$ to (x_1, x_3) and compute τ, σ_f and the ray parameter

q_1. Then we can explicitly evaluate

$$q_3 = \sqrt{\frac{1}{c^2(\xi_1, 0)} - q_1^2} \quad . \tag{40}$$

To obtain the reflectivity function, β, we introduce into (38) the factor (Bleistein, 1984b),

$$i \; \mathrm{sgn}(\omega) \; k/4 \; ,$$

and thereby obtain, by (17),

$$\beta(x_1, x_3) \sim \frac{8}{c(x_1, x_3)\sqrt{\pi}} \int d\xi_1 \sqrt{|\sigma_f|} \;\; q_3 \; \cdot$$

$$\int d\omega \sqrt{i\omega} \; F(\omega) \; e^{-2i\omega\tau(x_1, x_3, \xi_1, 0)} D(\omega, \xi_1) \quad . \tag{41}$$

IV. The Common Source Configuration

In this section, we show that the computations of the previous section allow us to obtain inversion formulae for the common source configuration and (by reciprocity) for the common receiver configuration. For a common source gather, we have

$$\underline{x}_s = \text{constant} , \tag{42}$$

so that $\tau(\underline{x}, \underline{x}_s)$ and

$$\underline{p}_s = \nabla\tau(\underline{x}, \underline{x}_s) \tag{43}$$

are independent of the surface parameters, $\underline{\xi}$. Denoting the ray direction to the receiver array by simply $\underline{p}$, (to facilitate comparison with the previous section), we have

$$\underline{p} = \nabla\tau(\underline{x}, \underline{x}_r) , \tag{44}$$

and Beylkin's determinant becomes

$$h(\underline{x}, \underline{\xi}) = \begin{vmatrix} \underline{p}_s + \underline{p} \\ \frac{\partial}{\partial \xi_1}\underline{p} \\ \frac{\partial}{\partial \xi_2}\underline{p} \end{vmatrix} . \tag{45}$$

Since determinants are linear functions of their rows, h can be written as $h = h_1 + h_2$ where

$$h_1 = \begin{vmatrix} \underline{p}_s \\ \dfrac{\partial}{\partial \xi_1}\underline{p} \\ \dfrac{\partial}{\partial \xi_2}\underline{p} \end{vmatrix} \tag{46}$$

and

$$h_2 = \begin{vmatrix} \underline{p} \\ \dfrac{\partial}{\partial \xi_1}\underline{p} \\ \dfrac{\partial}{\partial \xi_2}\underline{p} \end{vmatrix} \tag{47}$$

Since the last two rows of each of these determinants is the same as in the zero-offset case (with $\underline{x}_r$ taking the place of $\underline{x}_0$), we can proceed as at the beginning of the last section to obtain

$$h_1 = \frac{1}{p_3(\underline{x}, x_r)} \left[p_{s_3} p_3 + p_{s_1}^2 + p_{s_2}^2 \right] \frac{\partial(p_1, p_2)}{\partial(\xi_1, \xi_2)} \tag{48}$$

and

$$h_2 = \frac{1}{p_3(\underline{x}, \underline{x}_r)\, c^2(\underline{x})} \; \frac{\partial(p_1, p_2)}{\partial(\xi_1, \xi_2)} \quad . \tag{49}$$

Using the result of the appendix, we have

$$|h(\underline{x}, \xi)| = \left[p_{s_3} p_3 + p_{s_1}^2 + p_{s_2}^2 + \frac{1}{c^2(\underline{x})} \right] A^2(\underline{x}, \underline{x}_r) \cdot 16\pi^2 \sqrt{g_r}\; \hat{n}_r \cdot \underline{p}_r \tag{50}$$

and so, using the general result (21) we obtain

$$\alpha(\underline{x}) = \frac{4}{\pi}\iint d^2\xi \sqrt{g_r}\, \frac{A(\underline{x},\underline{x}_r)}{A(\underline{x},\underline{x}_s)} \left[1 + \frac{c^2(x)}{2} \quad p_{s_3}(p_3 - p_{s_3})\right] \hat{n}_r \cdot \underline{p}_r$$

$$\cdot \int d\omega\, F(\omega)\, e^{-i\omega[\tau(\underline{x},\underline{x}_s) + \tau(\underline{x},\underline{x}_r)]} D(\omega,\underline{x}_r) \; . \tag{51}$$

In (51) we have restated the result in (50) by use of the eikonal equation for $\underline{p}_s$.

The implementation of (51) is not as simple as the zero-offset case, the principle difficulty being the necessity to compute the two amplitude factors. In addition, the questions of constructing "synthetic apertures" in order to use an array of common source gathers as well as how to systematically use nearby common source gathers to reduce noise in a post migration processing remain open.

Obviously, the solution for the common <u>receiver</u> configuration can be derived at once from (51) by interchanging the subscripts s and r and defining

$$\underline{p} \;\; = \nabla\tau(\underline{x},\underline{x}_s) \tag{52}$$

$$\underline{p}_r = \nabla\tau(\underline{x},\underline{x}_r) \tag{53}$$

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A1: Appendix on the Zero-Offset Jacobian

It is natural to think of $\tau(\underline{x},\underline{x}_0)$ as the travel time along a ray path with initial point $\underline{x}_0$ and running point $\underline{x}$. In this model, $\underline{p}$ is the associated ray direction along the ray from $\underline{x}_0$ to $\underline{x}$. Furthermore, to solve the eikonal equation in this model by the method of characteristics (or "ray method"), one must consider the family of rays emanating from the "fixed" point $\underline{x}_0$ and consider $\underline{x}$ as an arbitrary field point. Of course, in constructing the eikonal, τ, and transport amplitude, A, we can consider $\underline{x}_0$ to be an arbitrary point on the observation surface, or otherwise put, we repeat the ray method solution for each $\underline{x}_0$.

On the other hand, if we think of the field point, $\underline{x}$, as fixed, it is equally valid to compute the travel time using a ray with initial point $\underline{x}$ and "final" point, $\underline{x}_0$. In this model, this "upward" ray direction is simply the negative of the ray direction on the "downward" ray at any given point on the ray, and we consider the family of rays emanating from the "fixed" point $\underline{x}$.

In the computation of the 2 by 2 Jacobian determinant in (31), we need to compute derivatives with respect to the second argument of $\underline{p}$. Thus we adopt the point of view where $\underline{x}$ is fixed. However, in our original description, the vector $\underline{x}_0$ was bound to the observation surface. Thus we introduce a new running variable, $\underline{y}$, governed by the ray equations,

$$\frac{d\underline{y}}{d\sigma} = \underline{p} \quad ; \quad \underline{y}(0) = \underline{x}$$

$$\frac{d\underline{p}}{d\sigma} = \frac{1}{2}\nabla\frac{1}{c^2} \quad ; \quad \underline{p}(0) \text{ free} \tag{A-1}$$

$$\frac{d\tau}{d\sigma} = \frac{1}{c^2} \quad ; \quad \tau(0) = 0$$

We impose no condition on the starting ray direction in (A-1) because, to construct the Green's function, we want the "conoidal" solution of the eikonal equation (Bleistein, 1984b). In order to impose the condition that each ray emanating from $\underline{x}$ passes through a point, $\underline{x}_0$, on the observation surface, we demand

$$y(\sigma_f) = \underline{x}_0(\underline{\xi}) \tag{A-2}$$

This last condition defines, for each such ray the final value, σ_f, of σ as a function of $\underline{\xi}$. (The question of having a unique ray to each surface point is subsumed in the issue of the non-vanishing of Beylkin's h determinant.)

We now evaluate our 2 by 2 Jacobian by introducing the ray parameter σ and then using the chain rule as follows:

$$\frac{\partial(p_1(\underline{x},\underline{x}_0),p_2(\underline{x},\underline{x}_0))}{\partial(\xi_1,\xi_2)} = \frac{\partial(p_1(\underline{x},\underline{y}),p_2(\underline{x},\underline{y}))}{\partial(\xi_1,\xi_2)}\Bigg|_{\sigma=\sigma_f} \tag{A-3}$$

$$= \frac{\partial(p_1,p_2,\sigma)}{\partial(\xi_1,\xi_2,\sigma)}\Bigg|_{\sigma=\sigma_f}$$

$$= \frac{\partial(p_1,p_2,\sigma)}{\partial(y_1,y_2,y_3)}\Bigg|_{\sigma=\sigma_f} \cdot \frac{\partial(y_1,y_2,y_3)}{\partial(\xi_1,\xi_2,\sigma)}\Bigg|_{\sigma=\sigma_f}$$

The second of these 3 by 3 Jacobians can be evaluated as

$$\frac{\partial(y_1,y_2,y_3)}{\partial(\xi_1,\xi_2,\sigma)}\Bigg|_{\sigma=\sigma_f} = \begin{vmatrix} \underline{t}_1 \\ \underline{t}_2 \\ -\underline{p}_0 \end{vmatrix} \tag{A-4}$$

$$= -\underline{p}_0 \cdot \underline{t}_1 \times \underline{t}_2$$

$$= -\underline{p}_0 \cdot \hat{n}\sqrt{g}$$

where we have introduced the surface tangent vectors

$$\underline{t}_i = \frac{\partial}{\partial \xi_i} \underline{x}_0 \quad , \ i = 1,\ 2\ , \tag{A-5}$$

and where $\underline{p}_0$ denotes the initial direction of the ray from $\underline{x}_0$ to $\underline{x}$ (or the negative of the final direction of the ray from $\underline{x}$ to $\underline{x}_0$) and finally $\hat{n}$ denotes the unit downward normal to the data surface at $\underline{x}_0$. The remaining Jacobian in (A-3) is the reciprocal of the ray Jacobian for rays emanating from $\underline{x}$

$$J(y) = \frac{\partial(y_1, y_2, y_3)}{\partial(p_1, p_2, \sigma)} \quad . \tag{A-6}$$

From the transport equation, we can derive the relation,

$$J(y) \quad A^2(\underline{y}, \underline{x}) \quad = \quad \text{constant} \tag{A-7}$$

(see (Bleistein, 1984b)). The constant in (A-7) can be evaluated by allowing $\underline{y}$ to approach the fixed field point $\underline{x}$ and using the constant c result with c being $c(\underline{x})$. We find for this limit,

$$\lim_{\underline{y} \to \underline{x}} JA^2 = \frac{1}{16\pi^2 p_3(\underline{x}, \underline{x}_0)} \quad , \tag{A-8}$$

so that the ray Jacobian is given by

$$J\Big|_{\sigma = \sigma_f} = \frac{1}{16\pi^2 p_3(\underline{x}, \underline{x}_0) A^2(\underline{x}, \underline{x}_0)} \tag{A-9}$$

Here we have used reciprocity to switch the arguments of A. Combining (A-3), (A-4), and (A-9) we find that the required 2 by 2 Jacobian is given by

$$\frac{\partial(p_1, p_2)}{\partial(\xi_1, \xi_2)} = 16\pi^2 p_3(\underline{x}, \underline{x}_0) A^2(\underline{x}, \underline{x}_0) \sqrt{g}\ \hat{n} \cdot \underline{p}_0 \tag{A-10}$$

as asserted in the text (32).

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ABSTRACT

The research group at the Center for Wave Phenomena has been developing inversion algorithms for progressively more complex background velocities and source-receiver configurations. In each extension, the crucial issue has been the determination of certain properties of a matrix involving derivatives of the travel time(s).

This report discusses and extends a recent result along these lines which appeared in a recent paper by Gregory Beylkin. Beylkin reduces the problem to consideration of a single canonical determinant, h, and then assumes that this determinant does not vanish. With this assumption, he establishes a very general inversion result. Consequently, future theoretical research will focus on the evaluation of h, on establishing conditions for its non-vanishing and on dealing with the phenomena arising when it does vanish.

Beylkin's paper uses powerful mathematical tools, such as the notions of pseudo-differential operators, generalized Radon transforms, and generalized back projections. Moreover, Beylkin frames his work in an N-dimensional space. Here, we dispense with much of this mathematical machinery and for convenience confine ourselves to the 3D case (and its 2.5D specialilzation). We are able to expound Beylkin's results by an approach similar to that presented earlier by Cohen and Hagin. However, we do not attempt to rigorously prove our results, but instead content ourselves with an intuitive derivation and the citing of Beylkin's main theorem.

We have established significant simplifications of Beylkin's result for the following cases of propagation governed by the acoustic wave equation: the zero-offset case and the common source offset gather and the common receiver offset gather with general c(x,y,z) reference velocity; and the case of common offset with a constant reference velocity.

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